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Transonic Small Disturbance Theory with Strong Shock Waves

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Introduction

THE most common methods of predicting aerodynamic characteristics at transonic speeds are either the transonic small disturbance (TSD) theory[1] or the full potential equation (FPE) theory.[2] The more accurate Euler equation solutions[3] are expensive to obtain, although for flows with strong shock waves such solutions are essential. The FPE theory requires that the flow is irrotational and treats the wing boundary conditions exactly (numerically). The TSD theory is an approximation to the FPE theory. One advantage of the TSD theory is the flexibility in deriving the approximate equation. This flexibility is generally utilized by a choice of a transonic scale parameter. The basic assumption of irrotationality in both these theories is only valid when the flow is shock free or contains only weak shocks. Both TSD and FPE solutions are in satisfactory agreement with realistic Euler equation solutions, provided that the basic restriction to weak shock waves is not violated. The thin wing boundary conditions can also introduce errors into the TSD solutions. If the flow has strong waves, however, then there is considerable disagreement among all three theories. Generally the predicted shock locations for the potential theories are much further aft than for the Euler equations. The problem addressed in this paper is to examine the error in the shock location in the TSD theory in two-dimensional flow and to derive a correction procedure within the confines of small disturbance theory. The basic hypothesis of the present theory is that the error in shock location is primarily due to the stronger shock strength predicted by TSD theory compared to that of the Euler equations. The technique uses two TSD solutions with different scaling parameters and an interpolation scheme derived for discontinuous transonic flows to give a corrected shock strength.

Analysis

The TSD equation for the perturbation velocity potential, $\phi(x,y)$, at a freestream Mach number M_∞, is

$$(1-M_\infty^2)\phi_{xx}+\phi_{yy}=(\gamma+1)M_\infty^q\phi_x\phi_{xx} \tag{1}$$

where γ is the ratio of specific heats and q is the transonic scaling parameter. The two most commonly used[4] values of q are 2 (Spreiter scaling) and 1.75 (Krupp scaling). The pressure coefficient $c_p(x,y)$ is

$$c_p(x,y)=-2\phi_x(x,y) \tag{2}$$

Associated with Eq. (1) are the usual tangency and far field boundary conditions. The weak shock jump conditions for Eq. (1) are

$$[\![[1-M_\infty^2-(\gamma+1)M_\infty^q\phi_x]\phi_x]\!] + \tan\phi_s [\![\phi_y]\!] \tag{3}$$

where $[\![\ ]\!]$ denotes a jump through a shock wave and ϕ_s is the angle between the shock normal and the x axis. For a shock normal to the freestream, the shock strength σ_T is defined as

$$\sigma_T=C_p^- - C_p^+ = -2(C_p^+ - C_p^*) \tag{4}$$

where C_p^+, C_p^- are the pressure coefficients just ahead of and behind the shock and

$$C_p^*=\frac{-2(1-M_\infty^2)}{(\gamma+1)M_\infty^q} \tag{5}$$

Consider now the Euler equation normal shock jump σ_E in terms of M_∞ and C_p^+ which is given[5] by

$$\sigma_E=\frac{2\gamma}{\gamma+1}(M_e^2-1)\left(\frac{2}{\gamma M_\infty^2}+C_p^+\right) \tag{6}$$

where the upstream shock Mach number M_e is given by

$$M_e^2=\frac{1}{(\gamma-1)}\left\{[2+(\gamma-1)M_\infty^2] \div\left[\frac{(\gamma-1)}{2}M_\infty^2C_p^+ + 1\right]^{\frac{\gamma}{\gamma-1}}-2\right\} \tag{7}$$

Equations (4) and (6) are shown for $M_\infty=0.755$ in Fig. 1, and it can be seen that as $|C_p^+|$ increases the discrepancy between σ_T and σ_E increases. Note that different transonic scalings not only give a different value of C_p^*, but generally a different value of C_p^+. Thus, for different scalings the shock strength may vary considerably.

The error in the shock location in the TSD solutions seems to be primarily due to the error in the shock strength as exhibited in Fig. 1. If the TSD equation is altered, still within its normal accuracy bounds, such that the shock jump approximates the Euler equation shock jump, then the resulting equation is a better compromise in representing the actual flow. The reason for this statement is that by matching the shock jump the new equation implicitly introduces an additional mechanism, formally negligible, that cancels the rotationality errors in a potential formulation.

If the correct shock strength is known and if two TSD solutions with different scalings are known, then a TSD solution with correct shock strength may be estimated using a linear combination of the known solutions. Thus, if σ_{T_1} and σ_{T_2} are the shock strengths of the TSD solutions, then a parameter ϵ can be found such that

$$\sigma_E=\sigma_{T_1}+\epsilon(\sigma_{T_2}-\sigma_{T_1}) \tag{8}$$

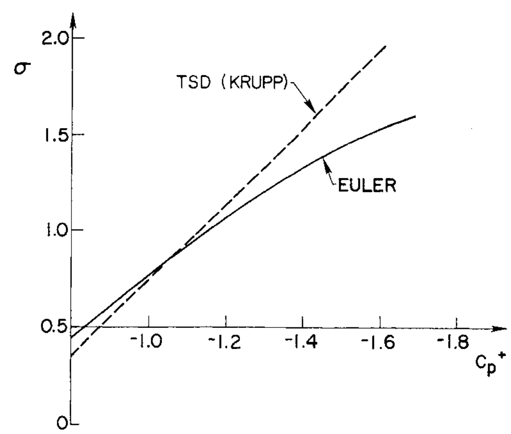

Fig. 1 Variations of normal shock strength with preshock pressure.

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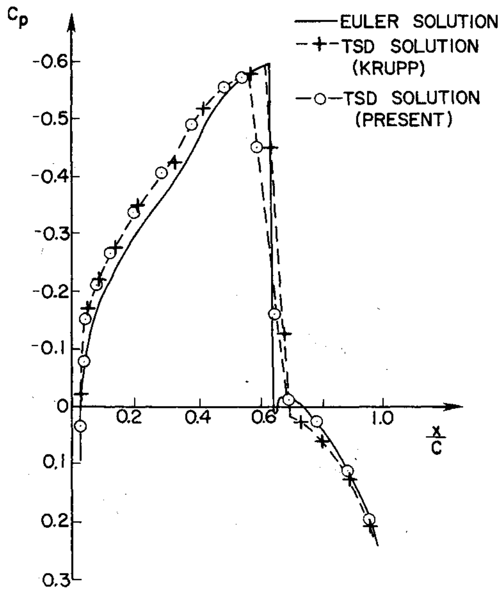

Fig. 2 Pressure distribution around a NACA 64A006 airfoil, $\alpha = 0$ deg, $M_\infty = 0.875$.

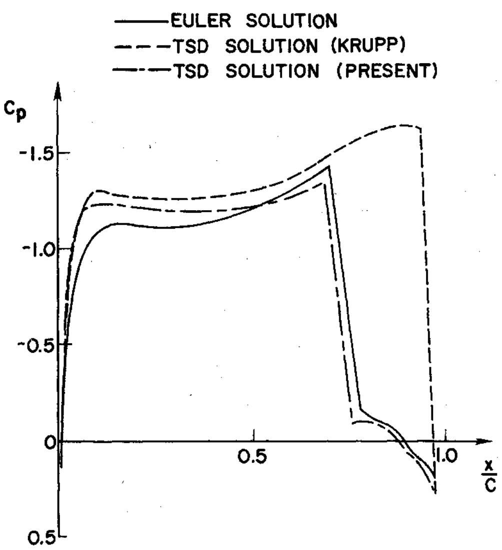

Fig. 3 Pressure distribution around the upper surface of a Korn airfoil, $M_\infty = 0.755$, $\alpha = 1.7$ deg.

where σ_E is the estimated Euler equation shock strength. If ϵ is known then the pressure distribution can be found using the strained coordinate method.[6] Thus,

$$C_p(x,y) = C_{p_1}(x',y) + \epsilon[C_{p_2}(\bar{x},y) - C_{p_1}(x',y)] \tag{9}$$

where $C_{p1}(x,y)$, $C_{p2}(\bar{x},y)$ are the pressure coefficients from the TSD solutions and

$$\bar{x} = x' + \delta x_{s0} x_1(x'), \quad x = x' + \epsilon(\bar{x} - x') \tag{10}$$

where δx_{s0} is the change in shock location between first and second TSD solutions. The function $x_1(x')$ is usually taken[6] to be

$$x_1(x') = \frac{x'(1-x')}{x_s(1-x_s)}$$

where x_s is the shock location for the first TSD solution.

The main problem now is to estimate the shock strength σ_E. This is given by Eq. (6) if C_p^+ is known. In TSD solutions Spreiter scaling gives good agreement[4] for moderately strong shocks and Krupp scaling gives good agreement for weak shocks. Hence it seems reasonable to take C_p^+ to be the average of the Spreiter and Krupp values of the preshock pressure coefficients. However, the justification for this choice really lies in the accuracy of the final results. Hence, given σ_E from Eqs. (6) and (7) the surface pressures can be found from Eqs. (8-10).

Results

The present method was first tested for a weak shock example, namely the flow over the NASA 64A006 airfoil at zero angle of attack and $M_\infty = 0.875$, to see if the present method would give the accurate Krupp scaling. This result is compared to an Euler equation result[7] in Fig. 2 and it can be seen that, apart from the shock capture characteristics, the present method does agree satisfactorily with both the Euler solution and the Krupp solutions. An example for a flow with a strong shock is shown in Fig. 3. In Fig. 3 the flow around a Korn airfoil at $M_\infty = 0.755$ and 1.7 deg angle of attack is compared to an Euler equation[8] solution and it can be seen that the agreement of the shock location predicted by the present method is satisfactory, although there is an error in the leading edge, which is almost certainly due to the use of thin airfoil boundary conditions in the TSD theory. Incidentally, if Spreiter scaling is used, the TSD result is almost coincidental with the present result.

Conclusion

A method to effectively choose a transonic scaling to place shock waves computed by the small disturbance theory at the location predicted by Euler equation solutions has been developed. The technique does effectively correct the shock location although discrepancies in the leading edge region still persist. This is probably due to the use of thin wing boundary conditions.

Acknowledgment

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